University Endowments in Seven Questions

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Executive Summary

- To a large extent, universities resemble alternative money managers: Shared characteristics include high asset volatility, leverage and exposure to liquidity risk.
- As a rule of thumb, university endowments can think about their risky allocations as the ratio of a risk premium divided by the variance of risky returns (adjusted by risk aversion).
- Non-endowment assets and liabilities are not adequately reflected in portfolio construction. Asset allocation can be materially different when the non-endowment balance sheet is factored in.
- Endowments may want to consider capital-efficient ways of combining active fixed income management with passive equity management.
- The optimal share of illiquid assets depends mostly on the liquidity risk premium and liquidity needs over a medium-term horizon.

In the 1925 Harold Lloyd classic film, "The Freshman," an intertitle describes a university as "a large football stadium with a college attached." Almost a century later, it is appropriate, perhaps, to call a leading university "a large alternative manager with a college attached." In this short paper about university endowments, we muse on fundamental questions of asset allocation, spending and time horizons.

We open the discussion by asking what's in a name.

QUESTION #1: ARE UNIVERSITIES CLOSET ALTERNATIVE MONEY MANAGERS?

The question may sound unwarranted. Consider the mission statement by James Tobin from Yale University: "The Trustees of endowed institutions are the guardians of the future against the claims of the present. Their task is to preserve equity among generations." This is hardly an alternative manager manifesto. The endowment serves a social purpose and helps with scholarships and tuitions. Most large universities depend on the return from their endowment to balance their operating budgets. What of the asset allocation? Figure 1 shows the asset allocation of an average endowment. Compared to a standard institutional portfolio, the allocation to alternative assets – hedge funds, venture capital and other private funds – seems aggressive at about 50%. However, this is a fully funded, directional allocation with no visible sign of the leverage or complex trading reminiscent of alternative managers.

Figure 1: Endowment breakdown by asset class

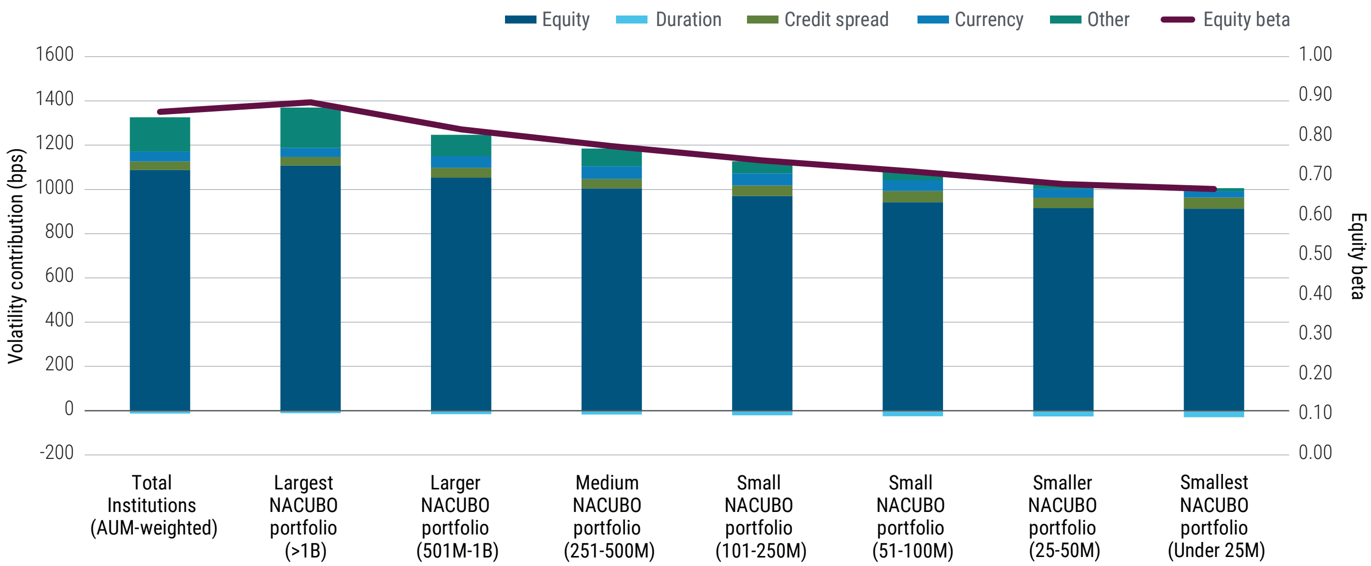
June 2020 - Percentages, AUM-weighted	
Public equity	35
Public fixed income	8
Public real assets	2
Hedge funds	19
Private equity, venture capital, other alternative assets	32
Cash	4
Total	100

Source: NACUBO as of June 2020

Yet, the alternative manager analogy is fitting for a number of reasons. Both endowment and hedge fund returns are directional, although to different degrees; the equity beta of an average hedge fund is about 0.4 – as much as half that of an endowment, as shown in Figure 2. The 12% volatility of endowment returns is significantly higher than a portfolio of hedge funds, even though most of it is contributed by the equity risk factor; and while there is no formal leverage in endowment balance sheets, leverage is embedded in their hedge fund and other alternative holdings, in addition to capital calls that act as hidden leverage during risk sell-offs. More importantly, the leverage is much higher on the liability-heavy university balance sheet than on the endowment balance sheet *per se*. Last but not least and contrary to received wisdom, both endowment and hedge fund after-fee returns depend to a large extent on beta, namely asset class selection. At best, the alpha explains a negligible part of performance. For example, in Dahiya and Yermack (2021), regression estimates in four-factor models (U.S. stocks, U.S. bonds, global stocks and hedge funds) of endowment portfolios find a statistically significant alpha of -39 basis points, with the alpha becoming more negative against a 60-40 mix of U.S. stocks and Treasuries. All in all, for the endowment-rich subset of universities, the answer to question number one is a qualified yes.

On a related note, it is worth pointing out that most universities have shunned macro funds, commodity hedge funds and quantitative hedge funds and have no exposure to insurance-linked securities. We often hear that volatility is irrelevant and that these strategies do not produce high enough returns. This is particularly surprising as universities are tax-exempt and should like high frequency strategies because the penalty of short-term versus long-term capital gains disappears. (One notable exception is the endowment of the Institute for Advanced Studies in Princeton, New Jersey – but its address is 1 Einstein Drive.)

Figure 2: University endowments: risk factor volatility contributions and equity betas



Note: Buckets are ranked by asset size (1 = largest, 7 = smallest)
Source: NACUBO and PIMCO as of June 2020

QUESTION #2: IS THERE AN ASSET ALLOCATION RULE OF THUMB?

A reasonable rule of thumb is the so-called Merton portfolio rule (see Merton (1969)). A simple expression of this rule states that, in a world of two assets – equity and cash – and an infinite horizon:

$$Equity\ allocation = \frac{\mu - r}{\gamma \sigma^2}, \tag{1}$$

where μ is the expected stock return, r the risk-free rate, γ the risk aversion index and σ the instantaneous volatility of the stock returns (see Appendix 1).

Consider the S&P 500 Index and cash. If the expected real equity yield is given by the cyclically adjusted earnings yield, about 2.75% at the time of writing, the expected real cash yield is -2.25% over the medium term, the risk aversion coefficient is 3 and the variance of equity returns is 3%, then the optimal asset allocation to the S&P is close to 56%.

The least measurable input in this expression is the risk aversion coefficient. An estimate of 3 is generally considered reasonable. Note that the risk allocation is very sensitive to this coefficient. Endowments are viewed as less risk-averse than average: A coefficient of 2 is compatible with the current endowment beta of about 80%. A risk-seeking coefficient of 1 (also known as growth optimal investing or Kelly rule) is low enough to violate the no-leverage constraint as it results in a 160% equity share (see Baz, Guo and Hakanoglu (2022)).

Assumptions behind the Merton rule of thumb – no transaction costs, constant yields, specific preferences – can be relaxed. Equally important are assumptions we make about the endowment model. This paper addresses several related questions: First, should we optimize the endowment portfolio or the university portfolio? If the former, all is simple. If the latter, then wealth should include non-endowment assets and liabilities. Second, the companion to Merton's asset allocation rule, called the Merton spending rule (see Merton, 1969), states that endowment spending should be proportional to wealth. Because wealth is highly volatile, so is consumption under this rule. How much should an endowment spend to remain solvent? Both of these questions are addressed in the following pages.

QUESTION #3: DO NON-ENDOWMENT ASSETS AND LIABILITIES MATTER?

It is hard to argue otherwise. The asset allocation should follow a total enterprise approach. Concretely, wealth should be defined as the expected net present value of the assets minus the liabilities. Assets include endowment assets and the expected present value of tuition fees and government and private donations. Liabilities include university debt and the expected value of operating and non-operating expenses.

To illustrate the total enterprise approach (see Merton (1993) for a similar example), consider a university with $3 billion in endowment assets, a net present value of $2 billion of non-endowment assets, such as tuition fees and donations and a net present value of $4 billion of (non-endowment) liabilities. The net wealth of the university is therefore $1 billion (3+2-4). A risk analysis shows that the net present value of the non-endowment assets is equivalent to 50% equity and 50% cash, whereas the net present value of the liabilities is equivalent to 25% equity and 75% cash. This means that the non-endowment balance sheet is equivalent to a zero equity position (50%*2-25%*4) and a short cash position of $2 billion (50%*2-75%*4). Now suppose that, in a separate exercise, the optimal allocation of net wealth to equity is found to be 80%, with a 20% optimal allocation to cash. In this case, the endowment should allocate $800 million to equity (80%*1) and $2.2 billion (20%*1+2) to cash. In an asset-only exercise, the endowment would have allocated $2.4 billion to equity and $600 million to cash – a far cry from the appropriate allocation.

This example illustrates the hidden mismatches on a university balance sheet. The discrepancy between an asset-only and a total enterprise approach is substantial. The reason for this is that non-endowment assets are more equity-sensitive than non-endowment liabilities. This is not unrealistic: University donations are likely to be more correlated to stock market returns than debt and expenses.

Of course, there is more to state variables than the stock market. For example, suppose alumni donations to a West Coast university are correlated to venture capital. In this instance, the allocation to venture capital in the endowment should be accordingly lower. Taking another example, if the Texan economy – and therefore the number of local students able to pay full tuition – is linked to the oil sector, then a Texan university with a local reach will want to consider an oil underweight in its endowment.

In general, the demand for risky assets is the sum of myopic demand and hedging demand. The myopic demand results from a narrow optimization; the hedging demand mitigates the risk of lower consumption from an adverse move in state variables (see Appendix 2).

QUESTION #4: SHOULD ENDOWMENTS SPEND THEIR EXPECTED RETURNS?

In John Campbell's words, "an endowment is a promise of vigorous immortality." (See Campbell, 2012.) A related question is: What is a spending rule that ensures immortality? A popular rule of thumb recommends that endowments consume their expected rate of return. We disagree. One of the curious properties of lognormal random walks is that a stock is very likely to trade below its expected value. It can be proved that the probability of an endowment return being higher than its expected return is:

$$p = \Phi\left(\frac{-\sigma\sqrt{T}}{2}\right), \tag{2}$$

where Φ is the cumulative normal operator, σ is the volatility of the endowment returns and T is the horizon (see Appendix 3).

As can be seen from the above equation, the probability of the spending rate being higher than the expected return goes to zero as the horizon goes to infinity. As Appendix 3 shows, for spending to be sustainable, meaning the endowment can sustain vigorous immortality, it must be lower than the expected return minus half the variance of returns:

$$s \leq \mu - \frac{\sigma^2}{2}. \tag{3}$$

If the expected arithmetic return μ of an endowment is 5% and its return volatility σ is 15%, then the spending rate should be lower than 3.875%.

Endowment spending is an intricate topic. One may wonder, for example, why endowments spend their resources on external assets (stocks, bonds and alternative assets) rather than on internal projects, such as science research. Spending on internal projects, it is sometimes claimed, is more in line with an endowment's mission – to increase the social rate of return and foster positive externalities. In addition to which, a stronger focus on, say, science research may in turn trigger more alumni donations.

Another interesting question is how to allocate endowment assets to ensure a minimum subsistence spending rate. For example, Dybvig (2000) uses a strategy similar to constant proportion portfolio insurance to protect spending. This portfolio value ends up being positively correlated with market momentum.

QUESTION #5: ACTIVE OR PASSIVE?

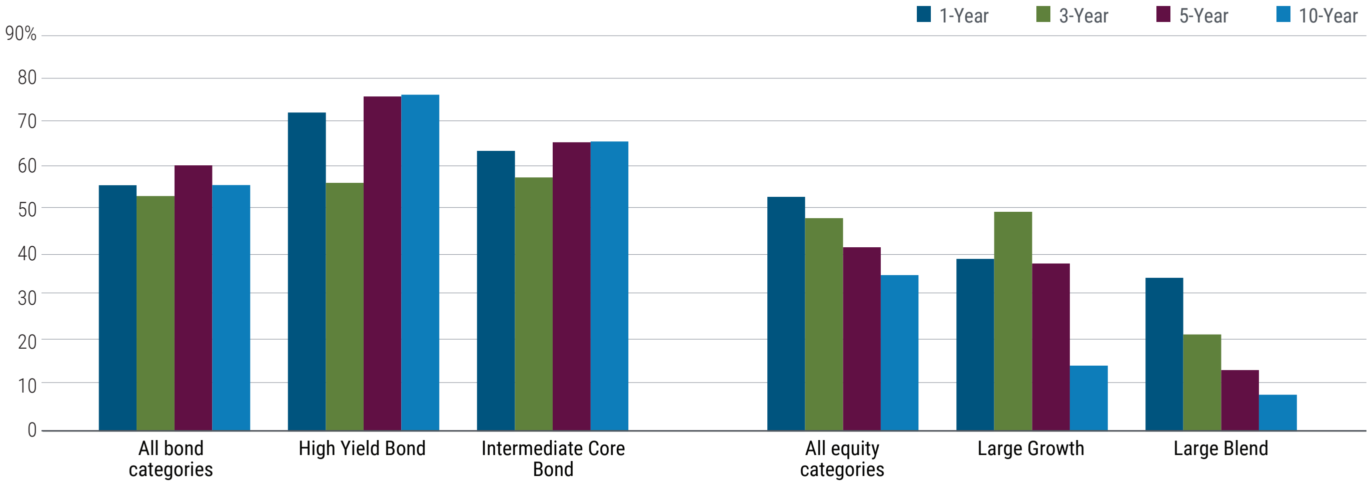
The straw man for the active debate stems from a simple argument by Bill Sharpe: If passive is defined as holding the market portfolio, and active is everything not passive, then it trivially follows that active managers as a whole, like passive managers, hold the market portfolio at any time. Both groups will therefore generate the same returns before fees. Because active fees are generally higher than passive fees, passive managers will outperform active managers, on average.

And indeed, active equity mutual funds have by and large underperformed their passive counterparts over the last 10 years. But, perhaps surprisingly, it turns out that active bond mutual funds have largely outperformed their passive bond counterparts, as shown in Figure 3.

Why should that be? And is there a flaw in Sharpe's argument? By focusing on two populations (active and passive) Sharpe may have neglected important differences within the active population. For example, there is more to active bond managers than active bond mutual funds: There are plenty of active *constrained* bond buyers. A case in point is central banks that own large proportions of their bond markets. Unlike active mutual funds, these central banks do not optimize a financial risk-return trade-off: They are highly constrained in that they buy bonds to influence GDP growth or the yield curve or foreign exchange or equity prices. *This constraint, which is costly like all constraints, can be viewed as an alpha transfer from constrained investors such as central banks to unconstrained investors such as mutual funds*. Note that constrained buyers are far more prevalent in bond markets than in equity markets.

What are the typical alpha strategies followed by bond mutual funds? To name a few, duration, yield curve, volatility and credit tilts; styles, such as carry, value and momentum; off-index investing in high yield, emerging markets and currencies; and cash-derivatives trades such as cash-CDS basis or Treasury basis. (For a detailed discussion, see Baz, Mattu, Moore and Guo (2017.))

Figure 3: Percentage of U.S. active mutual funds and ETFs that outperformed their median passive peers after fees (December 2020)



Source: Morningstar data and PIMCO as of December 2020
Charts are provided for illustrative purposes and are not indicative of the past or future performance of any PIMCO product.

How does this analysis affect the choice between active and passive managers in endowment public portfolios? To the extent these performance statistics are significant, it may be reasonable to combine active bond management and passive equity management in a portfolio. Better still, some equity investors may consider buying equity futures and investing the unencumbered cash in active bond funds to reap the benefits of both the equity risk premium and the alpha from active bond management.

QUESTION #6: WHAT IS THE OPTIMAL SHARE OF ILLIQUID ASSETS?

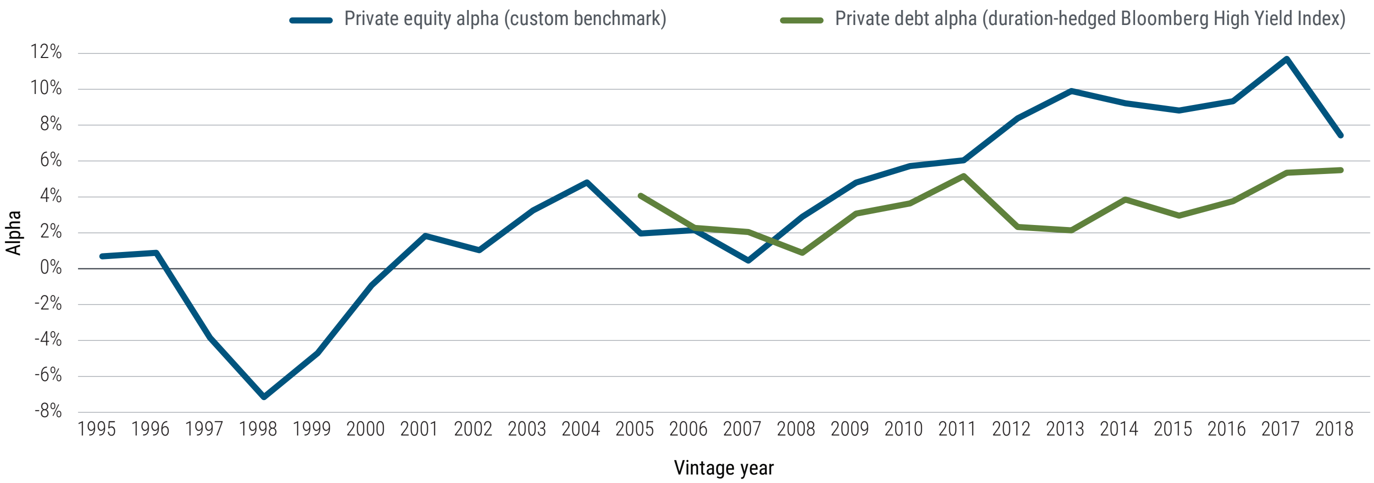
Investors with long horizons are naturally inclined to buy illiquid assets. And so it is with endowments. On paper, endowments have little sensitivity to short-term market gyrations and provide liquidity to the market in exchange for a liquidity premium. Of course, in times of distress, this model can be severely tested. In 2008, university endowments were hit simultaneously by market losses, lower donations, tuition fee pressures and capital calls. This in turn enticed endowments – perhaps against their best interests – into selling their private assets at deep discounts in thin secondary markets and incurring further losses in the process.

Therefore, the allocation to illiquid assets should largely depend on expected liquidity needs. This is the spirit of the portfolio model we discuss below.

We answer the asset allocation question in two steps: First, what is the expected return of illiquid assets – we focus here on private equity and private debt – and how is this expected return broken down into beta and alpha components? Second, what are the optimal illiquid holdings, given our return assumptions?

Looking at the Preqin dataset, the historical performance is 11.5% for private equity since June 1995, and 7% for private debt since December of 2007. To calculate a historical alpha, we need to assume appropriate benchmarks. We choose a custom value small cap leveraged benchmark for private equity and the Bloomberg Duration-Hedged US Corporate High Yield Index for private debt. As shown in Figures 4 and 5, private debt generated consistently positive alpha across vintages, albeit based on a limited sample, while private equity alpha appears to be more volatile. Because numbers related to vintages that have not paid off are less reliable, we calculate the average alpha for vintages with a low residual value to paid-in capital (less than 0.4). We find a high and statistically significant alpha of 3.1% for private debt and a lower and statistically insignificant alpha of 0.9% for private equity. Why such a difference in alpha? We can only conjecture that more crowding in private equity, more opportunities from bank regulation and a tame volatility environment favored private debt relative to private equity.

Figure 4: Private equity and private debt alpha



Source: Preqin as of December 2020

Charts are provided for illustrative purposes and are not indicative of the past or future performance of any PIMCO product.

Figure 5: Private equity and private debt average alpha for RVPI < 0.4

For RVPI < 0.4	Private equity	Private debt
Average alpha*	0.9%	3.1%
STD of alpha*	3.6%	1.8%

Source: Preqin and PIMCO as of December 2020

We now turn to expected returns and asset allocation. We assume an investor with an 80-20 allocation to equity and bonds in its public portfolio (not unlike an endowment). The question is: What is the investor's optimal allocation to illiquid assets? Based on our expected returns in Figure 6, we calculate optimal illiquid holdings based on investors' required liquidity over a 5-year horizon. For example, if an investor needs 50% of assets under management to be liquid with probability higher than 90% over the next five years, then the optimal private equity and private debt allocations should be 16.5% and 14.3%, respectively. The illiquid allocation is then close to 30%. If the liquidity needs are 70% of assets under management, then, all else being equal, private equity and private debt allocations drop substantially to 1.5% and 1.6% respectively, or about 3% of total in illiquid assets.

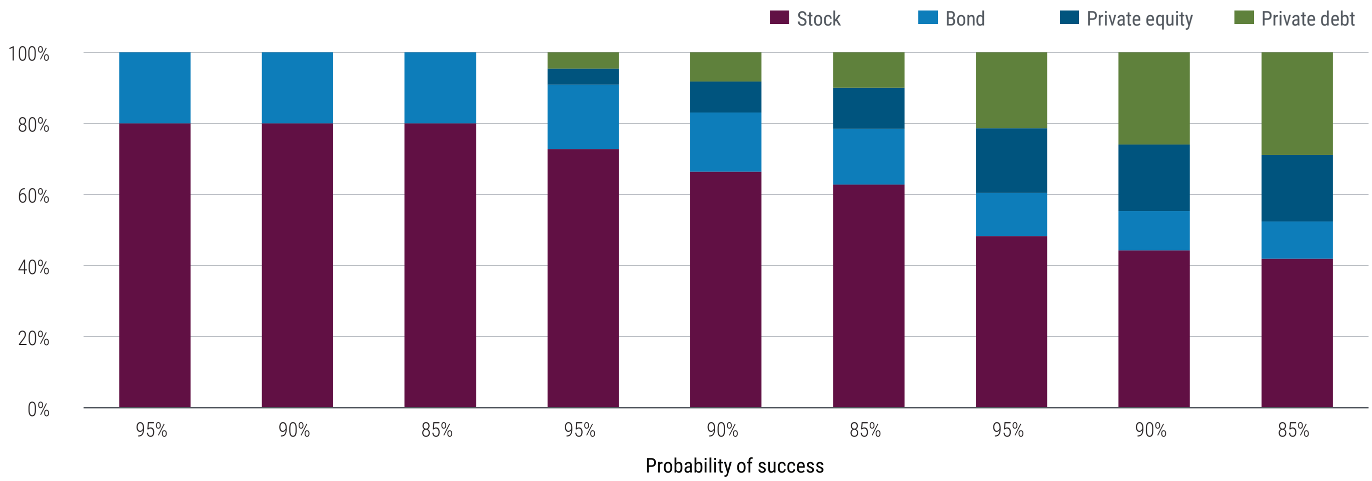
Figure 6: Expected returns and liquid/illiquid asset allocation for an 80-20 Investor

	Expected return
S&P 500	5.4%
U.S. Agg	1.2%
Private equity	8.1%
Private debt	6.0%

Private equity		Probability of success	
Required liquidity	95%	90%	85%
80%			
70%		1.5%	4.0%
60%	4.5%	8.8%	11.5%
50%	13.3%	16.5%	17.5%
40%	18.3%	18.8%	18.8%

Private debt		Probability of success	
Required liquidity	95%	90%	85%
80%			
70%		1.6%	4.5%
60%	4.6%	8.2%	10.0%
50%	11.3%	14.3%	17.0%
40%	21.4%	25.9%	28.9%

Source: PIMCO as of 30 June 2021
For illustrative purposes only.
Expected return is an estimate of what investments may earn on average over the long term and is not a prediction or a projection of future results. Actual returns may be higher or lower than those shown and may vary substantially over shorter time periods. There is no guarantee that these returns can be realized.



Source: PIMCO as 30 June 2021. **Hypothetical example for illustrative purposes only.** There can be no assurance that the investment approach outlined above will produce the desired results or achieve any particular level of returns. No representation is being made that any account, product, or strategy will or is likely to achieve profits, losses, or results similar to those shown. Hypothetical or simulated performance results have several inherent limitations. Unlike an actual performance record, simulated results do not represent actual performance and are generally prepared with the benefit of hindsight. There are frequently sharp differences between simulated performance results and the actual results subsequently achieved by any particular account, product or strategy. In addition, since transactions have not actually been executed, simulated results cannot account for the impact of certain market or financial risks such as lack of liquidity or the ability to withstand losses. There are numerous other factors related to the markets in general or the implementation of any specific investment strategy, which cannot be fully accounted for in the preparation of simulated results and all of which can adversely affect actual results.

The implications of this asset allocation model are clear: Some university endowments are so large (e.g., Harvard, Yale, Princeton) that they can absorb large shocks without much impact on their operational expenditure. The required liquidity as a proportion of their assets under management is small. They can thus afford a large allocation to illiquid assets.

Here again, not all happens according to plan. As is well-known, endowment investment committees are often comprised of wealthy trustees with strong views. Given the attractive performance of venture capital over the past 20 years and the preponderance of West Coast graduates on West Coast endowment investment committees, one would expect significant outperformance in West Coast university endowments. Yet, data – albeit limited – do not confirm this.

In a conversation on illiquid assets, we would be remiss if we omitted the impact of smoothing private returns. One is always tempted to think that marking-to-market is a sacrosanct dogma of financial life. Yet, there is something to be said for smoothing. In a world of long-term mean reversion of returns, long horizon investors such as endowments experience much-reduced accounting drawdowns. For example, assuming a 12% expected return, the expected maximum drawdown over three years goes from 40% under an unsmoothed 24% volatility, to 12% under a smoothed volatility of 10%. If drawdowns provide the trigger for forced selling, as is often the case, then smoothing becomes a way of fighting our behavioral demons and avoiding the proverbial selling at a low (see Baz, Davis and Hahn (2022)).

QUESTION #7: WHY THIS PAPER NOW?

And who needs to read it when the year-to-date performance of major endowments has been stellar?

Figure 7: YTD Performance - Selected University Endowments

Endowment	Date	Current fiscal year return (%)
University of Chicago	11/09/21	37.6
Harvard University	10/14/21	33.6
Massachusetts Institute of Technology	10/14/21	55.5
University of Pennsylvania	09/28/21	41.1
Princeton University	10/29/21	46.9
Stanford Management Co.	10/26/21	40.1
Yale University	10/15/21	40.2

Source: Pensions and Investments

The table above is eloquent. Yet, things are never as good or as bad as they seem. Indeed, stellar past performance decreases expected rates of return just like poor performance increases them. In markets where valuation ratios of most risky assets are living in the right tails and expected returns in the left tails, it pays off to re-examine basic assumptions.

On all points raised above, we welcome an open conversation.

ACKNOWLEDGEMENTS

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APPENDIX 1: MERTON'S ASSET ALLOCATION AND SPENDING RULES

Consider the consumption problem:

$$V = maxE \int_t^{\infty} e^{-\beta t} \frac{C^{1-\gamma}}{1-\gamma} dt \quad \text{(A.1)}$$

subject to:

$$dW = [(r + \alpha(\mu - r)W - C]dt + \alpha W \sigma dB, \quad \text{(A.2)}$$

where V is the value function, β is the time preference parameter, C is consumption, γ is the risk-aversion index, W is the total wealth, r is the risk-free rate, μ is the instantaneous expected return on the risky asset, σ is the instantaneous volatility of the risky asset return, t is time and B is a Wiener process.

By the Bellman principle, we have:

$$\beta V = max_{\alpha,C}\{\frac{C^{1-\gamma}}{1-\gamma} + V_W[(r + \alpha(\mu - r)W - C]\} + \frac{V_{WW}}{2}\alpha^2 W^2 \sigma^2. \quad \text{(A.3)}$$

Taking first-order conditions, guessing $V = KW^{1-\gamma}$ and replacing in the above equation, we get the optimal asset allocation:

$$\alpha^* = \frac{\mu - r}{\gamma \sigma^2}. \quad \text{(A.4)}$$

As well as the optimal spending rule:

$$C^* = \frac{1-\gamma}{\gamma}\left[\frac{\beta}{1-\gamma} - r - \frac{1}{2\gamma}\left(\frac{\mu - r}{\sigma}\right)^2\right]W. \quad \text{(A.5)}$$

Example: With μ=4%, r=1%, β=2%, γ=3 and σ=15%, we have an allocation to the risky asset of α*=44% and consumes 1.8% of wealth per year. Note that the expected instantaneous return on the portfolio is $(44\% * 4\%)$+$(56\% * 1\%)$=2.32%, which is higher than the spending rate.

APPENDIX 2: ENDOWMENT HEDGING DEMAND

In a model with n risky assets and a state variable S, it can be shown that asset demand (see Merton, 1973) is:

$$w^* W = -\frac{J_W}{J_{WW}}\Sigma^{-1}(\mu - r) - \frac{J_{WS}}{J_{WW}}\Sigma^{-1}\sigma_S, \quad \text{(A.6)}$$

where w^* is the vector of asset demands, J is the value function, μ is the vector of asset expected instantaneous returns, Σ is the covariance matrix for asset returns and σ_S is the volatility of the state variable S.

The $-\frac{J_W}{J_{WW}}\Sigma^{-1}(\mu - r)$ term is called the myopic demand, while the $-\frac{J_{WS}}{J_{WW}}\Sigma^{-1}\sigma_S$ term represents hedging demand as explained below.

Assume Σ is diagonal. Then it can be shown that $-\frac{J_{WS}}{J_{WW}} = -\frac{C_S}{C_W}$, where C_S and C_W are the partial derivatives of consumption with respect to the state variable and wealth, respectively.

If C_S>0, then a positive change in the state variable would cause higher consumption and

$-\frac{J_{WS}}{J_{WW}} = -\frac{C_S}{C_W} < 0$. Think of the state variable as the oil price or real estate. Then, because consumption increases with the state variable, the investor will buy less of an asset if it is positively correlated with that state variable.

APPENDIX 3: SPENDING RATE AND EXPECTED RETURN

If an endowment consumes its expected instantaneous return, then the probability of sustainability at horizon T is:

$$p = Prob\left(We^{\left[\mu - s - \frac{\sigma^2}{2}\right]T + \sigma\varepsilon\sqrt{T}} > W\right), \quad \text{(A.7)}$$

where W is the initial wealth, μ is the instantaneous real rate of return, s is the instantaneous real spending rate, σ is the volatility of endowment real returns and ε is a standard normal variable.

When s=μ, then taking logarithms, we find that:

$$p = \Phi\left(\frac{-\sigma\sqrt{T}}{2}\right), \quad \text{(A.8)}$$

as stated in the text. For long time horizons, the probability of exceeding the initial endowment goes to zero.

$$\text{When } s = \mu - \frac{\sigma^2}{2}, \text{ then } p = \Phi(0) = \frac{1}{2}. \quad \text{(A.9)}$$

Past performance is not a guarantee or a reliable indicator of future results. All investments contain risk and may lose value. Investing in the bond market is subject to risks, including market, interest rate, issuer, credit, inflation risk, and liquidity risk. The value of most bonds and bond strategies are impacted by changes in interest rates. Bonds and bond strategies with longer durations tend to be more sensitive and volatile than those with shorter durations; bond prices generally fall as interest rates rise, and low interest rate environments increase this risk. Reductions in bond counterparty capacity may contribute to decreased market liquidity and increased price volatility. Bond investments may be worth more or less than the original cost when redeemed. **Equities** may decline in value due to both real and perceived general market, economic and industry conditions. **Corporate debt securities** are subject to the risk of the issuer's inability to meet principal and interest payments on the obligation and may also be subject to price volatility due to factors such as interest rate sensitivity, market perception of the creditworthiness of the issuer and general market liquidity. Investing in **foreign-denominated and/or -domiciled securities** may involve heightened risk due to currency fluctuations, and economic and political risks, which may be enhanced in emerging markets. **High yield, lower-rated securities** involve greater risk than higher-rated securities; portfolios that invest in them may be subject to greater levels of credit and liquidity risk than portfolios that do not. The **credit quality** of a particular security or group of securities does not ensure the stability or safety of the overall portfolio. **Sovereign securities** are generally backed by the issuing government. Obligations of U.S. government agencies and authorities are supported by varying degrees, but are generally not backed by the full faith of the U.S. government. Portfolios that invest in such securities are not guaranteed and will fluctuate in value. **Derivatives** may involve certain costs and risks, such as liquidity, interest rate, market, credit, management and the risk that a position could not be closed when most advantageous. Investing in derivatives could lose more than the amount invested.

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